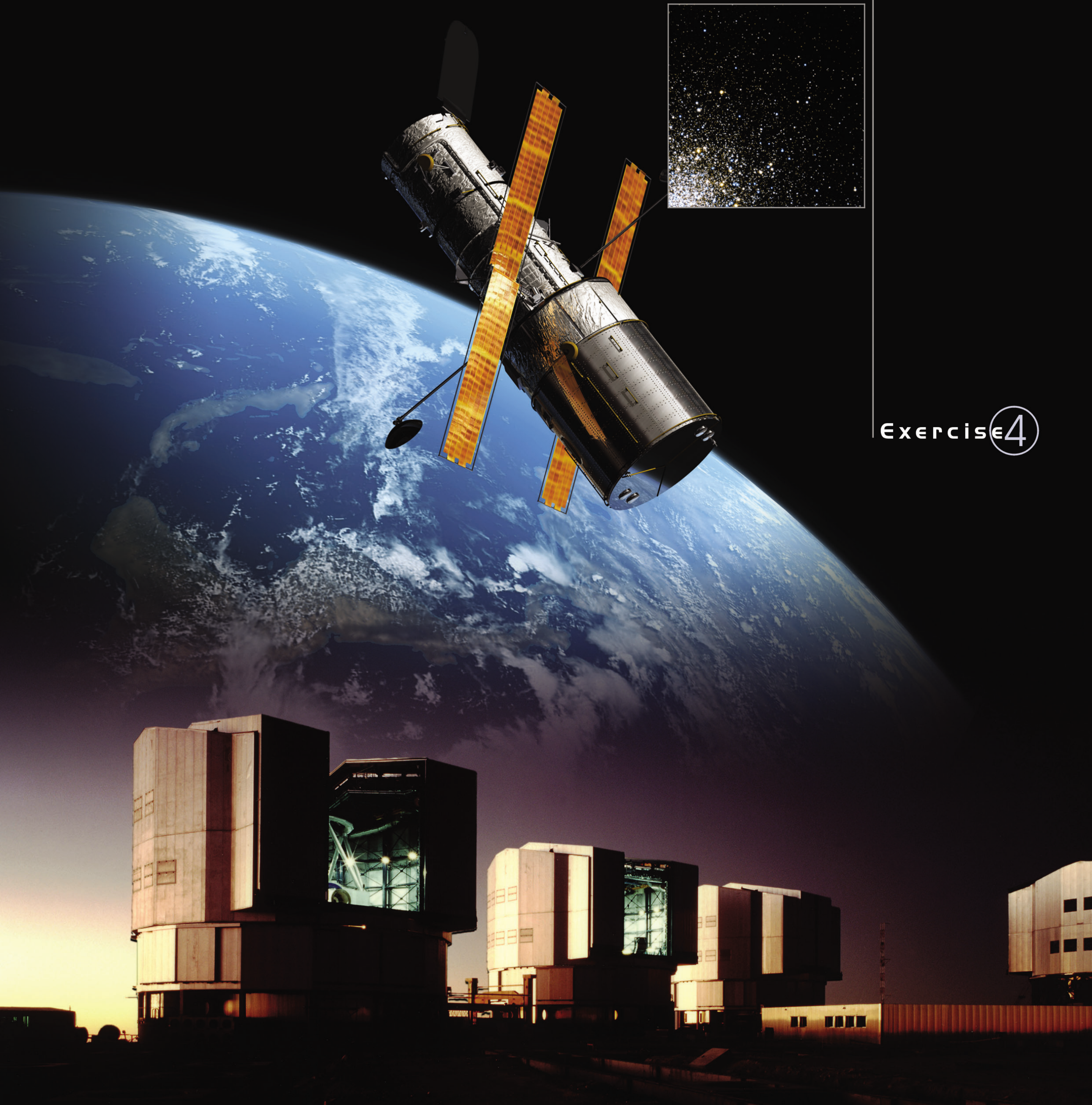
THE ESA/ESO ASTRONOMY EXERCISE SERIES

Student exercises in astronomy using observations from the NASA/ESA Hubble Space Telescope and the ESO telescopes



Exercise 4

Measuring a Globular Star Cluster's Distance and Age

Based on Observations with the ESO Very Large Telescope

esa

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The ESA/ESO Astronomy Exercise Series 4

Measuring a Globular Star Cluster's Distance and Age

Astronomy is an accessible and visual science, making it ideal for educational purposes. Over the last few years the NASA/ESA Hubble Space Telescope and the ESO telescopes at the La Silla and Paranal Observatories in Chile have presented ever deeper and more spectacular views of the Universe. However, Hubble and the ESO telescopes have not just provided stunning new images, they are also invaluable tools for astronomers. The telescopes have excellent spatial/angular resolution (image sharpness) and allow astronomers to peer further out into the Universe than ever before and answer long-standing unsolved questions.
The analysis of such observations, while often highly sophisticated in detail, is at times sufficiently simple in principle to give secondary-level students the opportunity to repeat it for themselves.

This series of exercises has been produced by the European partner in the Hubble project, ESA (the European Space Agency), which has access to 15% of the observing time with Hubble, together with ESO (the European Southern Observatory).

Figure 1: The ESO Very Large Telescope
The ESO Very Large Telescope (VLT) at the Paranal Observatory (Atacama, Chile) is the world's largest and most advanced optical telescope. With its supreme optical resolution and unsurpassed surface area, the VLT produces very sharp images and can record light from the faintest and most remote objects in the Universe.

Introduction

Stars

A star is a giant ball of self-luminous gas with physical properties such as mass, temperature and radius. Also of interest to astronomers is the distance from the star to Earth. The closest — and most-studied — star is, of course, our own Sun.

Hydrogen burning

The light emitted by most stars is a by-product of the thermonuclear fusion process in the stars inner core. A normal sun-like star is composed of about 74% hydrogen and 25% helium, with the remaining 1% being a mixture of heavier elements. The most common fusion process in sun-like stars is 'hydrogen burning', where four hydrogen nuclei fuse into one helium nucleus. The process occurs over several stages, illustrated in Fig. 2. In the first step of the process two protons fuse to form deuterium, a form of heavy hydrogen. This is a very rare event, even at the star's dense core, where the temperature is a few million degrees. This is why all sun-like stars do not explode in a wild runaway reaction when starting the fusion process, but remain in this stable phase of the star's life for several billion years. While the star is stable its surface temperature, radius and luminosity are nearly constant. The nuclear reactions at the core generate just enough energy to keep a balance between the outward thermal pressure and the inward gravitational forces.

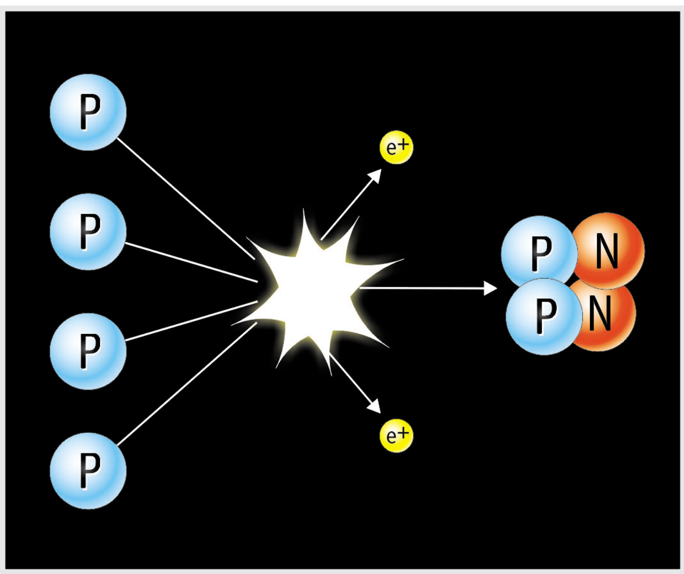

Figure 2: Hydrogen burning
The simplest form of energy 'production' in stars takes place by the fusion of four hydrogen nuclei into one helium nucleus. The process has several steps, but the overall result is shown here.

The mass of a helium atom is only 99.3% of the mass of the four original hydrogen nuclei. The fusion process converts the residual 0.7% of mass into energy — mostly light. The amount of energy can be calculated from Einstein's famous equation, $E = \boldsymbol{M}c^2$. As c^2 is a large number, this means that even a small amount of matter can be converted into an awesome amount of energy. The residual 0.7% of the mass of four hydrogen nuclei involved in a single reaction may seem tiny, but when the total number of reactions involved in the fusion process is considered, there is a substantial total mass (and thus energy) involved.

Star Clusters

The term 'star clusters' is used for two different types of groups of stars: open star clusters and globular star clusters.
Open star clusters are loose collections of a hundred to a few thousand relatively young stars. These are typically a few hundred million years old, a fraction of the few billion years that stars take to evolve. These clusters are found in the disc of our Galaxy, the Milky Way and often contain clouds of gas and dust where new stars form. The typical diameter of an open star cluster is about 30 light-years (10 parsecs).

Globular clusters — the oldest structures in the Milky Way

A few hundred compact, spherical clusters called globular clusters exist in the disc and halo of our Milky Way and are gravitationally bound to our Galaxy.

Each globular cluster consists of a spherical group of up to a million stars and is typically 100 light-years across. Most of the globular clusters are very old and most likely predate the formation of the Galaxy that took place about 12 billion years ago when

Figure 3: The Pleiades (Messier 45) in the constellation of Taurus
This is one of the most famous star clusters in the sky. The Pleiades can be seen with the naked eye from even the most light-polluted cities. It is one of the brightest and nearest open clusters. The Pleiades cluster contains more than 3000 stars, is about 400 light-years away and only 13 light-years across (courtesy Bruno Stampfer and Rainer Eisendle).

the majority of the proto-galactic material settled into the disc.

Many globular clusters have probably been destroyed over the past billions of years by repeated collisions and interactions with each other or with the Milky Way. The surviving globular clusters are older than any other structures in our Milky Way.
The astrophysical study of globular clusters

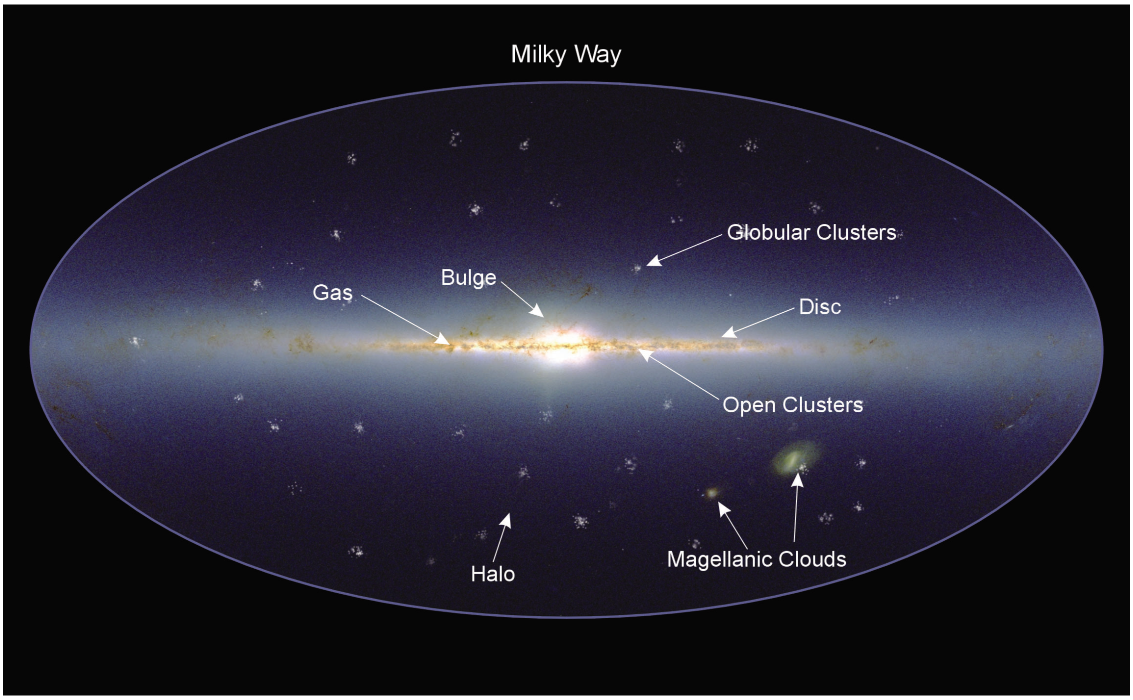

Figure 4: The Milky Way
This illustration gives an overview of the Milky Way galaxy. The different components of this complicated system of stars, gas, and dust are marked. The plane of the disc lies along the central horizontal line. The globular clusters are distributed in a spherical halo around the galactic centre. It is believed that this distribution is related to the fact that these clusters of stars formed early on in the history of the Galaxy.

forms an important part of the research interest of the international astronomical community. These clusters of stars are significant, not only as valuable test beds for theories of stellar structure and evolution, but also because they are among the few objects in the Galaxy for which relatively precise ages can be determined. Because of their extreme longevity they provide a very useful lower limit to the age of the Universe. The distribution of their ages and the correlation between the age of a cluster and its chemical abundance makes these systems an invaluable probe into the processes of galaxy formation.

All stars gathered in a globular cluster share a common history and differ from each other only in their mass. Therefore, globular clusters are ideal places to study the evolution of stars. In the following exercises, you will determine some properties of one particular globular cluster, Messier 12.

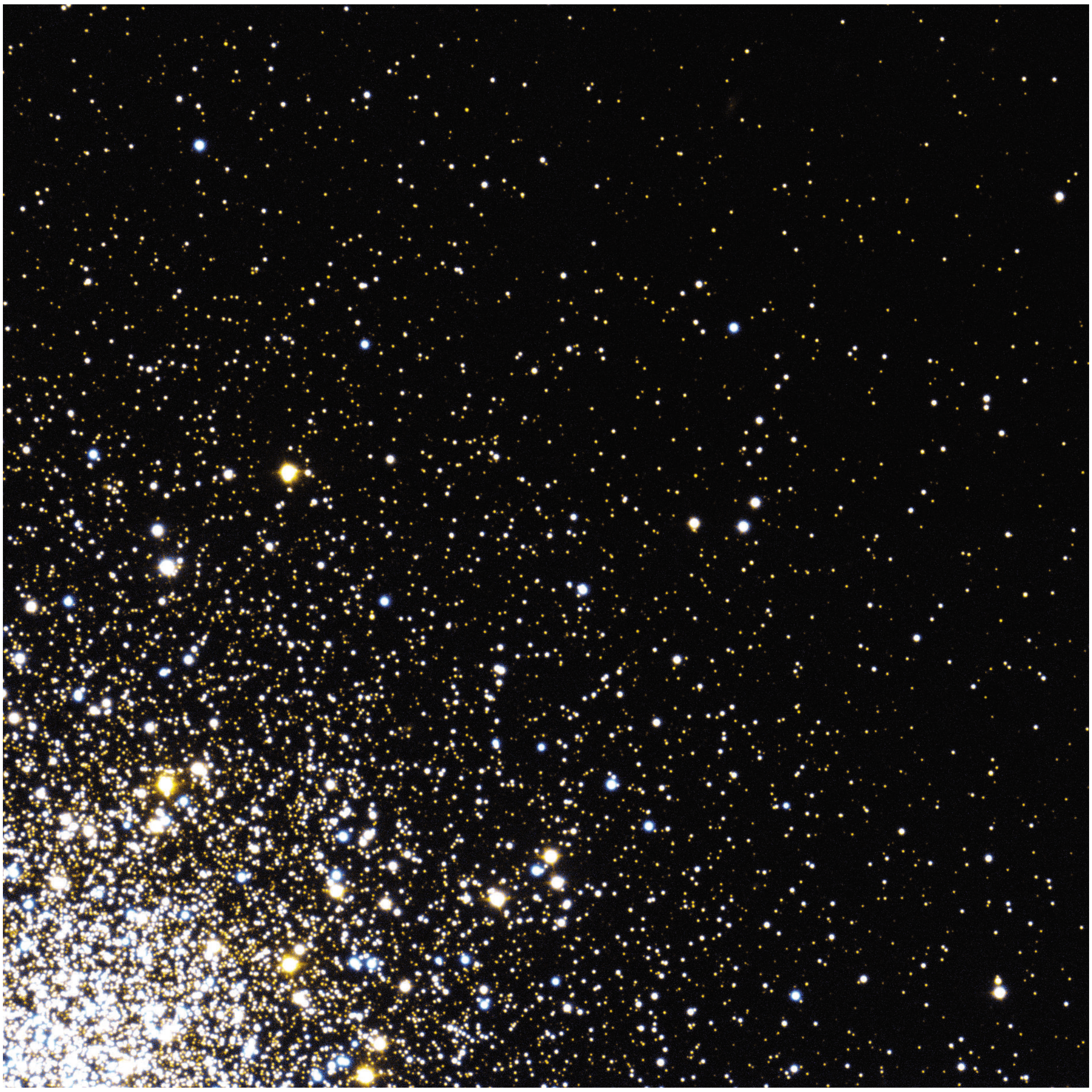

Figure 5: The outer region of globular cluster M12
This two-colour image was constructed from observations made through a blue (B) and through a green (V) filter using ESO's Very Large Telescope (VLT). The B image is shown in blue and the V image as red in this composite image. Some of the stars are clearly brighter in the B image (seen as bluish stars) while others are brighter in the V image (seen as yellowish stars).

The Globular Cluster Messier 12

The globular cluster Messier 12 (or M12), also called NGC 6218, was discovered in 1764 by Charles Messier and thus became the 12th Messier object. Like many other globular clusters, Messier described it as a 'Nebula without stars' a consequence of the modest resolving power of his telescopes. William Herschel was the first to resolve the cluster into single stars in 1783.

M12 is located in the constellation of Ophiuchus and can be seen with binoculars from places with very low light pollution. The visible magnitude of the whole globular cluster is 6.7 (read about magnitudes in the Astronomical Toolkit, page 2) and the brightest star in the cluster has a visible magnitude of 12.

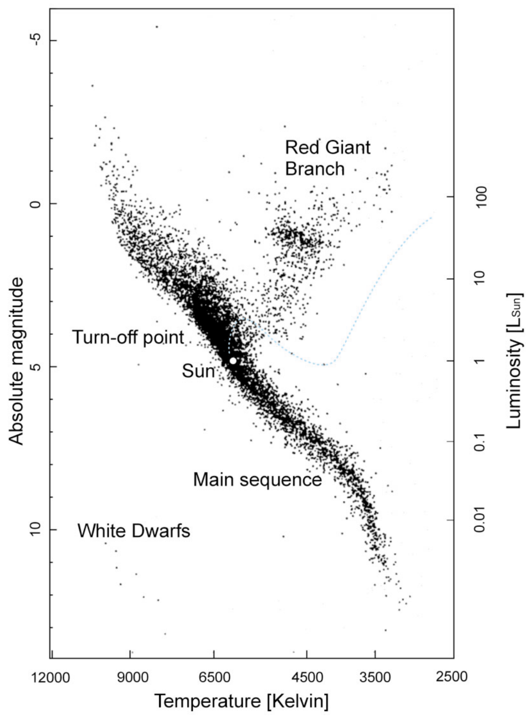

Figure 6: A Hertzsprung-Russell Diagram of nearby stars
The H-R diagram shows the relationship between surface temperature and luminosity of the stars. Note the prominent Main Sequence and the different regions where red giants and white dwarfs dominate. The location of the Sun is marked as well as the 'route' that a star of one solar mass will follow during the different phases of its life.
The position of the Sun on the diagram is determined by its surface temperature of 5800 K and its absolute magnitude of +4.8.

The NGC (New General Catalogue) was published in 1888. It lists open and globular star clusters, diffuse and planetary nebulae, supernova remnants, galaxies of all types and even some erroneous entries corresponding to no objects at all.

The Hertzsprung-Russell diagram

A graph showing luminosity L (or absolute magnitude M) against surface temperature T for stars is called a Hertzsprung-Russell diagram (short: H-R diagram). Fig. 6 shows a general example which has been constructed from observations of stars in nearby clusters where the distances are known (from HIPPARCOS measurements). The surface temperature of a star T can be derived from measured values of its colour (m_B-m_V) (see the Astronomical Toolkit).

It is clear from looking at the H-R diagram that the (L, T) measurements for different stars form a curious pattern when plotted on the diagram. The stars are concentrated in specific areas (marked in the figure). The H-R diagram holds the key to understanding how stars evolve with time. Different stars will – depending on their mass – move through the diagram along specific routes.

Stellar evolution in the H-R diagram

Stars spend most of their life on the Main Sequence, burning hydrogen slowly in a state of stable equilibrium. This is obviously why most stars are observed to lie on the Main Sequence, approximately a straight line from the upper left to the lower right in the diagram. When the hydrogen supply in the core of the star is depleted, hydrogen burning is no longer possible. This ends the main sequence phase of the star's life and the equilibrium of gas pressure and gravitational contraction in the stellar core is no longer stable. Hydrogen fusion now takes place in a surrounding shell while the core starts to shrink. As the core contracts the core pressure and the central temperature rise, so that helium nuclei in the core begin to fuse and

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form heavier elements. This cycle can be repeated using progressively heavier elements as each lighter element is exhausted in the core. During this phase the star appears as a red giant. Such stars appear on the H-R diagram off the main sequence line to the upper right. The higher central temperature causes the outer shells of the star to expand and cool down and thus the surface temperature falls. The whole star becomes very large and, because of the lower surface temperature, it mainly emits radiation of longer wavelengths out into space so the star looks red.
Despite their low surface temperature T, all red giants have a high luminosity, L, because of their huge radius, R. This results from Stefan-Boltzmann's radiation law for blackbody radiation:

$$L = \sigma 4\pi R^2 T^4$$

where σ is the Stefan-Boltzmann constant. Typical values for red giants are $R \sim 10^2\ R_{sun}$, $T \sim (3..4)10^3$ K , so L is about $10^3\ L_{sun}$.

When the advanced fusion processes in the stellar core can no longer be sustained, the core collapses again. Once again the temperature of the core increases and now the outer shells of the star are expelled. A so-called planetary nebula is formed from the remnants of the star's shell (see ESA/ESO Astronomy Exercise 3). The collapsed core is very hot (white) and the star is very small. Such a star is very suitably called a white dwarf and is the end of a normal sun-like star's life.

To make a rough estimate of the relationship between luminosity L and surface temperature T for all the main sequence stars, let us look at the H-R diagram (Fig. 6). The approximate straight line of the Main Sequence spans about one power of ten in temperature: $(3 \times 10^3 \ldots 3 \times 10^4)$ K. The range of luminosities spans about six powers of ten: $(10^{-2} \ldots 10^4)\ L_{sun}$. We can therefore roughly estimate: $L \propto T^6$ for the main sequence stars.

To give some examples:

- A high mass star on the main sequence with a surface temperature of about $T_{star} = 1.0 \times 10^4$ K has a luminosity of about
  $L_{star} = (10/5.8)^6 \cdot L_{sun}$,
  or approximately 26 times the Sun's luminosity. (The Sun's luminosity has a standard value

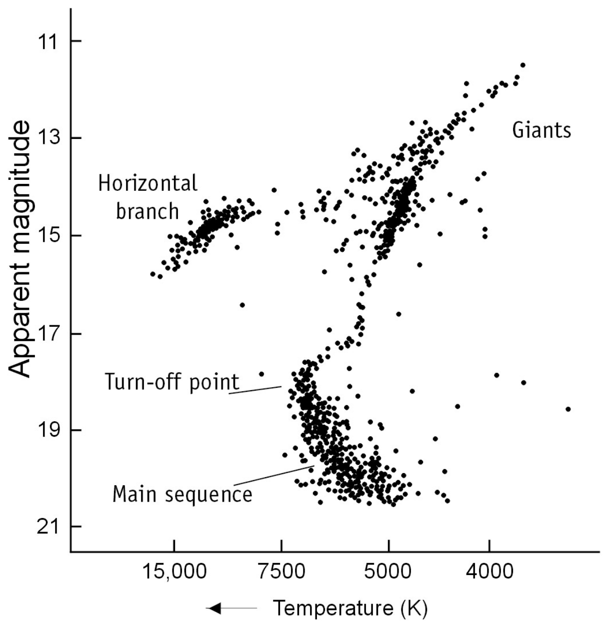

Figure 7: Typical Hertzsprung-Russell Diagram of a globular cluster
After billions of years of evolution a globular cluster H-R diagram shows a short Main Sequence (MS) in the lower right part. An area called the Red Giant Branch starts from the MS and reaches toward the upper right of the diagram. The point where the MS branch and the Red Giant Branch connect is called the turn-off point.

of 1 on the luminosity scale).

- A low mass star with $T_{star} = 3.5 \times 10^3$ K has a luminosity of only about 5% of the Sun's luminosity.

B-V colour index: a clue to the surface temperature

All the information we can extract from the stars is contained in the radiation that we receive from them. As explained in the Astronomical Toolkit, different filters and colour-systems can be used to measure the brightness of a star. In this exercise we use a B-image and a V-image. In your analysis of these images you will find the apparent m_B and m_V magnitudes of a sample of stars in the cluster. Then you can calculate the m_B–m_V values (the B–V colour index). Finally you will be able to determine the surface temperature of the stars (see Astronomical Toolkit).

For a cluster, a H-R diagram is the key

A cluster is a group of stars. The life of a cluster is determined by the lives of the different types of stars within it.
For a globular cluster, observations have shown that very little gas and dust remain, so new stars are rarely born in such a cluster. The stars we see in a globular cluster are all 'adults' and have evolved in different ways depending on their mass.

Most low-mass stars are settled on the Main Sequence. This is because low mass stars are expending their energy very slowly. They burn their hydrogen reserves quietly and will continue doing so for billions of years. They will therefore stay on the Main Sequence for a long time.

On the contrary, the heavier stars in the cluster have already converted the hydrogen in their cores and become red giants. This all happened long ago, so today no high-mass, hot stars remain to fill the upper half of the Main Sequence (see Fig. 7). These stars are now located in the diagonal area that starts from the Main Sequence and reaches out towards the upper right of the diagram known as the Red Giant Branch.

The point where the Main Sequence and the Red Giant Branch meet is called the turn-off point and is an important clue to the age of the cluster. In the following exercise, you will measure the co-ordinates of this point on your diagram and determine the age of M12.

Observations, data reduction and analysis

The globular cluster M12 was observed on June 18th, 1999 using the FORS1 instrument on ANTU (UT1) of the VLT at the ESO Paranal Observatory (Chile). For this exercise we have chosen images of the outer parts of the cluster where there are slightly fewer stars. The exposures were taken through a blue filter (B-band) and through a green filter (V-band for Visual).

To observe and to reduce data (the process of removing instrumental and other artefacts from the data) is a job requiring large telescopes and sophisticated computer programs. The really interesting part for astronomers — the data analysis — starts afterwards.

In this exercise the data has already been collected and reduced. We have simplified the analysis a little by selecting a set of stars that can be considered as representative of the population of the whole cluster.

Hints for analysing the images

To analyse the images, the B and V magnitudes of each star have to be measured carefully. Errors made in the first parts of this exercise will affect the later results.

The 45 stars are split into six sections:

1 Five stars nos. 1 to 5 — ‘training stars’

2 Four stars nos. 6 to 9 — ‘calibration stars’

3-6 The remaining stars are split into four groups (A, B, C and D) to reduce the work and give you enough time to make precise measurements.

To make your measurements as accurate as possible we suggest the following procedure:

- Put the gauge (see Figs. 8–9 bottom) on the star and shift it back and forth. Find where the values are too high and too low. Then move the gauge midway between these values and read off your measurement. Repeat this a few times and take the mean value.
- Let different people in each group measure each star at least twice and take the mean value of these measurements.
- Between measuring each star repeat the gauge training to make sure that you measure in a consistent way from star to star.

Task 1 B-Band training

For the training stars (nos. 1 to 5), the magnitudes are given in the table (Fig. 10).

? Use them to train yourself to use the gauge by making measurements on the B-image (Fig. 8) and comparing them with the table. Make sure you get the same results.

Task 2 B-band calibration

Each group should measure the calibration stars (no. 6 to 9) independently. The measurements can then be calibrated with the results from other groups.

? Measure the calibration stars in the B-image (Fig. 8), add these numbers to the table and compare your results with the other groups. If there are differences, have a look at those stars and at the training stars again.

Task 3 B-band magnitudes

? Measure the blue magnitude (m_B) of each labelled star in the area you have been assigned (A, B, C or D) in Fig. 8 and add the measurements to the table.

Task 4 V-Band training

? Train yourself by making measurements on the V-image (Fig. 9) and comparing them with those given in the table. Make sure you get the same results.

Task 5 V-band calibration

? Measure the calibration stars in the V-image (Fig. 9), fill in the table and compare your results with the other groups. If there are differences, have a look at those stars and at the training stars again.

Task 6 V-band magnitudes

? Use Fig. 9 to determine the green magnitude (m_V) of each labelled star in the area you have been assigned (A, B, C or D). Add these values to the table.

Task 7 Colour-Index

? Calculate the m_B-m_V value for each star and add the results to the table.

Task 8 Surface Temperature

? Use the diagram, Fig. 3 in the Astronomical Toolkit, to convert the m_B–m_V values into values of surface temperature, T, for the stars and add the results to the table.

Task 9 H-R Diagram

The main sequence of the Hyades cluster has been plotted as a reference on the diagram (Fig. 11). Note that the absolute magnitude, M_V, has been measured for the Hyades.

? Plot the measured apparent magnitudes (m_V) versus the calculated surface temperature (T) corresponding to the M12 stars on the same diagram.

Task 10 Main Sequence fitting: Distance-modulus

For the stars in M12 we now know (m_V, T), and from the reference Hyades measurements we know (M_V, T) for a standard Main Sequence. The distance modulus (see Astronomical Toolkit) of M12 is the shift in the vertical axis between the two main sequences you have plotted.

? Calculate the distance modulus m_V-M_V for M12.

Task 11 Distance to M12

? Use the distance modulus and the distance equation (see the Astronomical Toolkit, if necessary) to determine the distance D of M12.

Task 12 Extinction correction

The distance you have just found is not quite correct since our Galaxy contains a lot of gas and dust that weakens the starlight that comes from behind (and from within). The dust and gas also colour the starlight reddish due to a process known as Rayleigh scattering (that works most efficiently for short waved light i.e. bluish light). These two processes are known under the name 'interstellar extinction'.

We would like you to correct for the part of the extinction that weakens the light (making the magnitude of the observed stars too high and the calculated distances therefore too large)[1]. The corrected distance modulus m-M is:

$m-M-A$,

where A is the extinction correction factor. The distance equation changes slightly due to this:

$D=10^{(m-M-A+5)/5}$

For M12, A is given by Harris et al. to 0.57 magnitudes (in the V band, which is what we use to measure m-M).

? Calculate a new distance that has been corrected for interstellar extinction.

? Is the corrected distance very different from the not corrected one found in task 11?

? Discuss the differences and discuss the implication of this correction (one of many that astronomers use in their daily life) on our general understanding of the size of the Universe.

[1]This is as mentioned a simplification since there also is some smaller extinction influence on the B-V (or temperature) term.

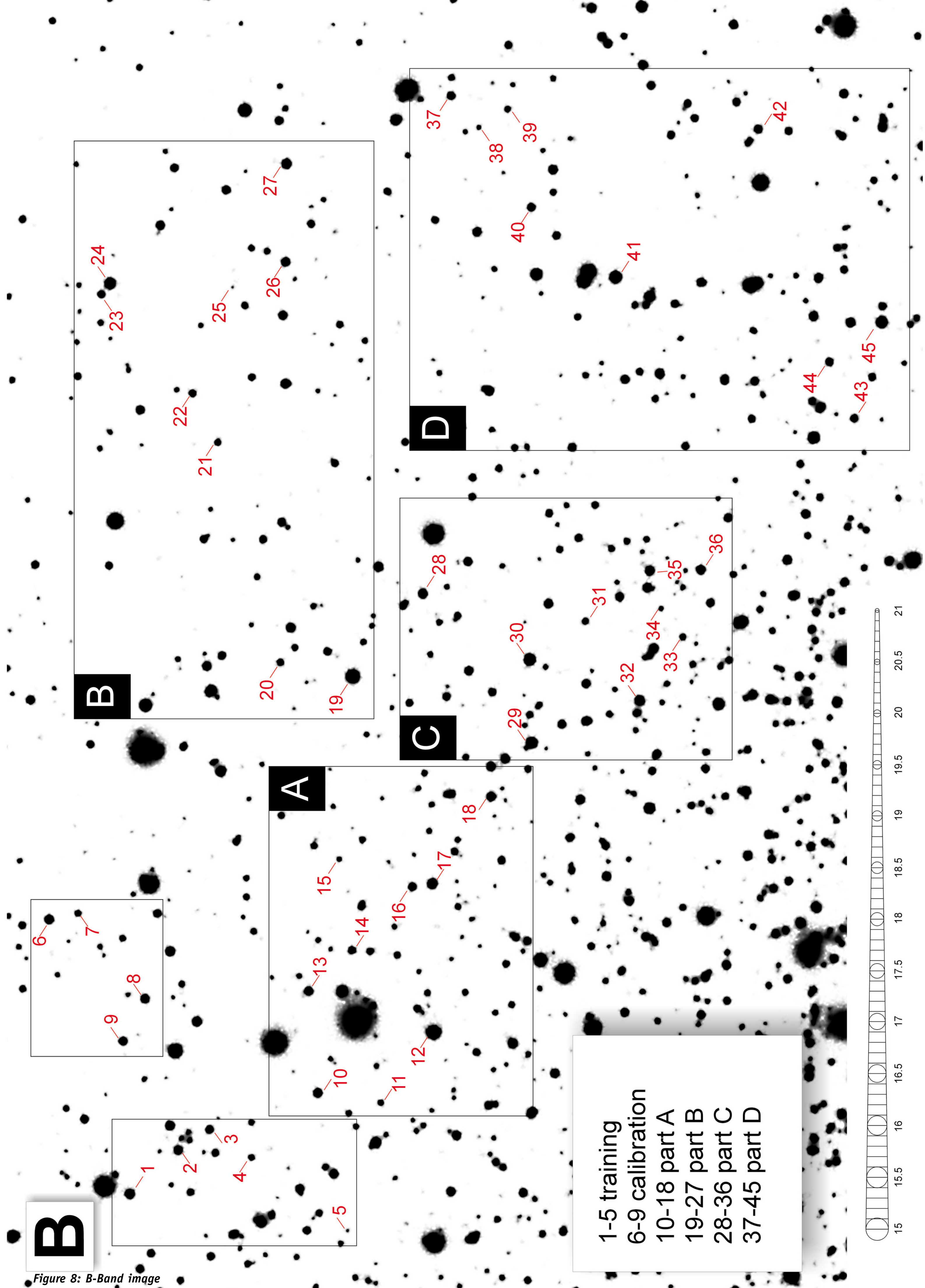

Figure 8: B-Band image

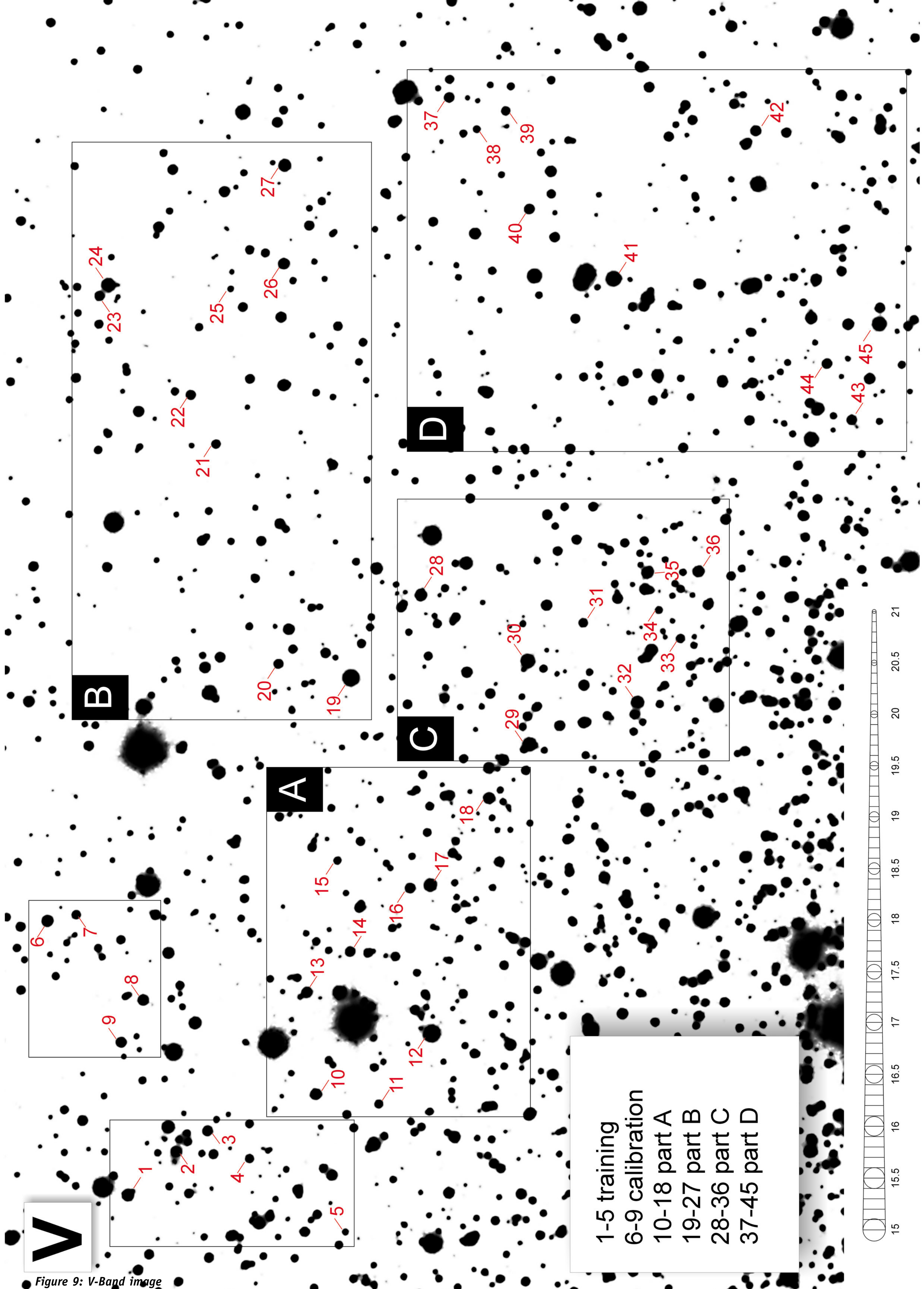

Figure 9: V-Band image

Tasks

Figure 10: Table of values

	Scientists' Values				ESA/ESO's measurements/calculations			
Star	**B**	**V**	**B-V**	**T**	**B**	**V**	**B-V**	**T**
1	18.82	17.98	0.84	5250	18.70	17.90	0.8	5403
2	19.02	18.31	0.71	5744	19.00	18.20	0.8	5403
3	19.32	18.65	0.67	5864	19.30	18.70	0.6	6122
4	19.96	19.25	0.71	5699	19.90	19.10	0.8	5403
5	21.05	20.21	0.84	5265	21.00	20.10	0.9	5076
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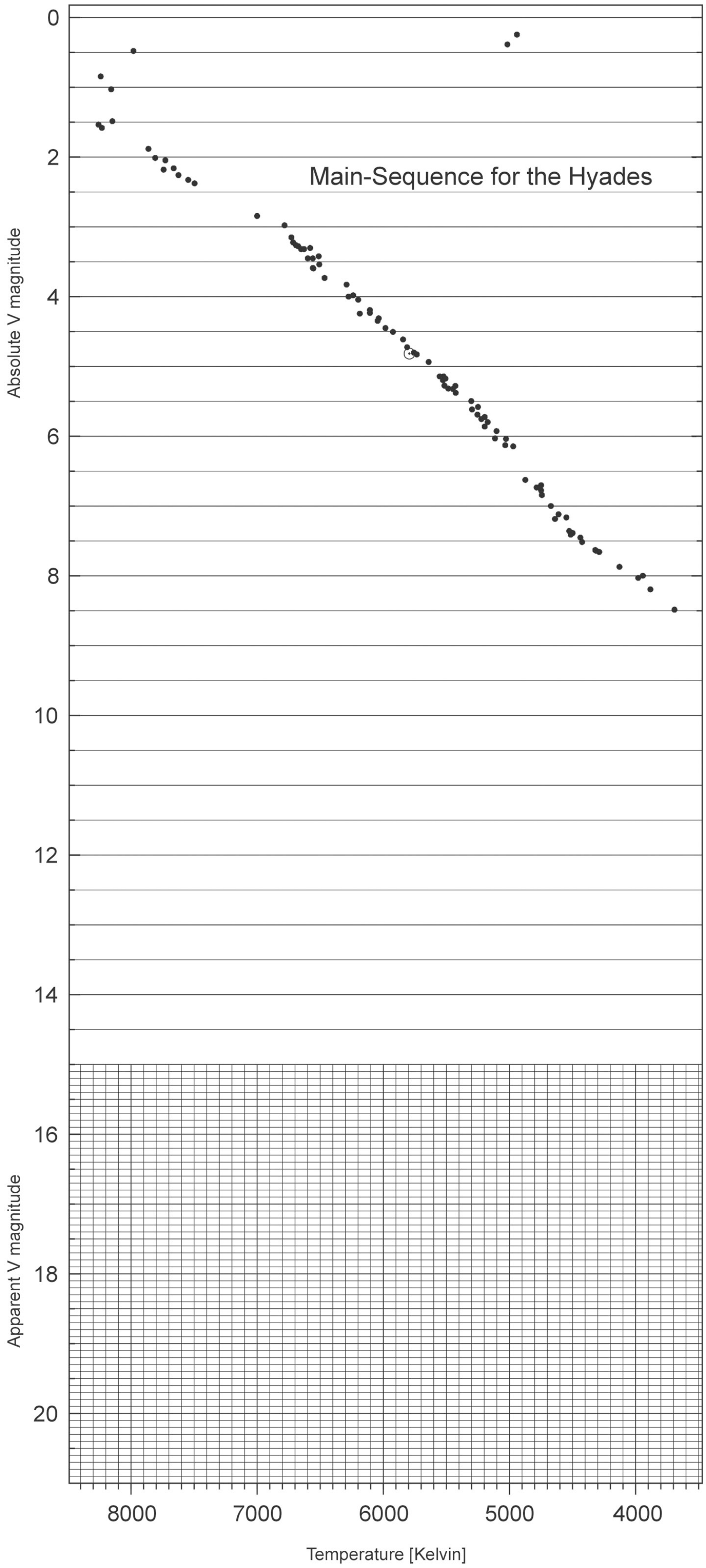

Figure 11: Plotting diagram
The results of the measurements in tasks 1–9 are to be plotted here.
The calibrated Main Sequence for the Hyades cluster was observed with ESA's HIPPARCOS satellite (from de Bruijne et al., 2001).

Tasks

Task 13

Scientists have ealier calculated a distance to the cluster of D = 4900 parsecs from the original versions of a larger sample of data. If your answer differs by less than 20% from this value, you have made very accurate measurements, thorough calculations and you can be very proud of your work!

If your result has a larger error, there could be several reasons. Some possibilities are:

- Are your measurements of the magnitudes precise enough?
- Can you think of different more sophisticated methods to analyze the data and fit to the Main Sequence?
- Think of other ways to improve your results.

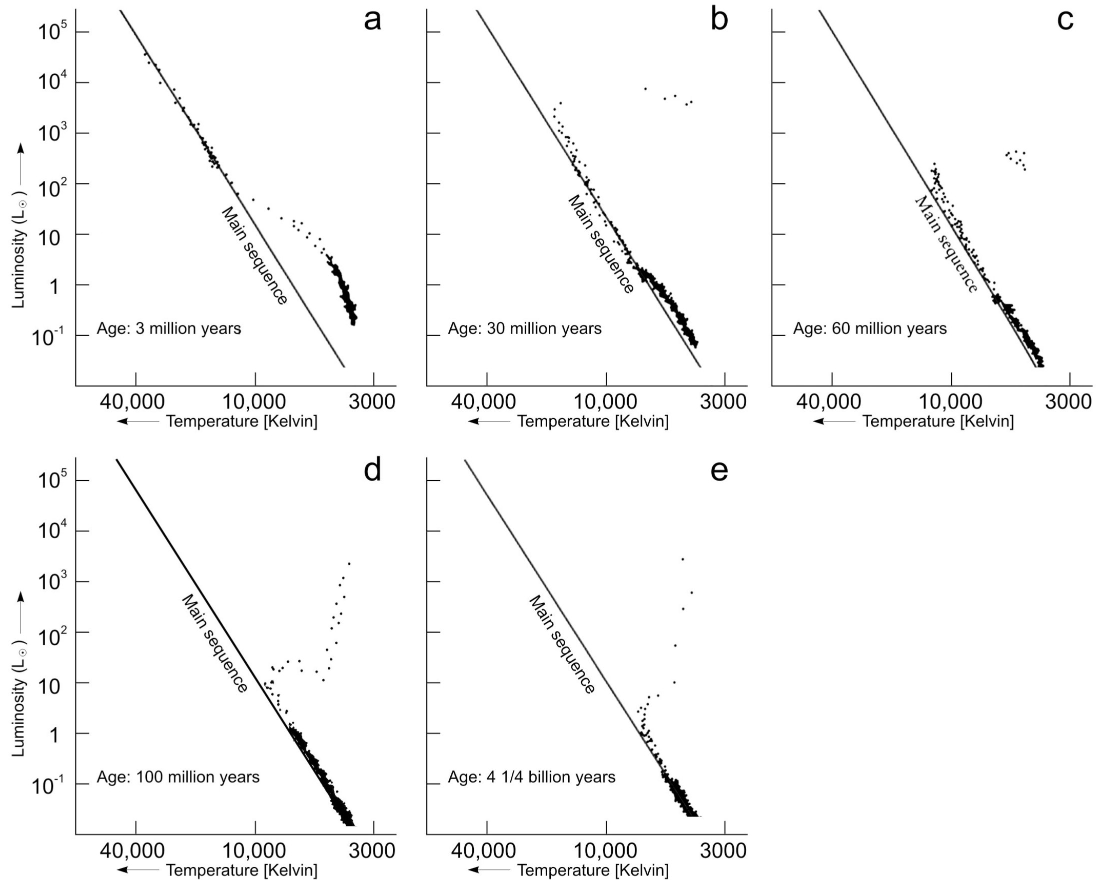

Figure 12: The Evolution of a Theoretical Globular Cluster
This series of H-R diagrams was created by computing how the equations of stellar evolution affect a sample of stars over time. In 12a the biggest and most luminous stars are on the main sequence (T > 10 000K) and the smaller stars are still beginning the hydrogen burning process (low temperature, low luminosity).
In 12b the biggest stars have consumed most of their core hydrogen fuel and are burning the shell reserves. Their luminosity has decreased and become redder, they have moved away from the Main Sequence, the Red Giant branch has started to appear and the turn-off point is visible. No hot and luminous stars remain on the upper part of the Main Sequence.
In 12c-e the upper part of the Main Sequence is almost deserted, while the Red Giant Branch is more heavily populated. The lower part of the Main Sequence indicates a large population of solar mass stars with surface temperatures in the range of 4000 to 8000 K. These stars will remain in this phase for billions of years (adapted from R. Kippenhahn).

Evolution of globular clusters

The shape of the Main Sequence is basically the same for all globular clusters, whatever their age. The Main Sequence fitting method used above could also be used for other clusters of different ages to determine their distances in the same way.
However, observations of H-R diagrams of different clusters show that the upper part of the Main Sequence changes shape depending on the cluster's age (see Fig. 12). In older clusters the most luminous stars of the cluster have evolved and moved to the Red Giant Branch. As a result the upper part of the Main Sequence becomes shorter and the connection between the Main Sequence branch and the Red Giant Branch (the turn-off point) moves down, much as a candle burns down with time.
Consequently, we may infer that the position of the turn-off point is an important clue in determining the age of the cluster.

Task 14 Turn-off point: from magnitudes to luminosity

? Determine the apparent magnitude of a star at the turn-off point of M12. Calculate the luminosity of this star relative to the Sun's luminosity using the formula given in the Astronomical Toolkit.

Turn-off point: from luminosity to mass

Once the luminosity is known, we can determine the mass of the star using the 'mass-luminosity' relation. For stars on the Main Sequence there is an observed correlation between mass and luminosity, where luminosity and mass are expressed relative to the values for the Sun ($L_{sun} = 4 \times 10^{26}$ W, $\boldsymbol{M}_{sun} = 2 \times 10^{30}$ kg):

$$L = \boldsymbol{M}^{3.8}$$

Task 15

? Convert the luminosity derived in Task 14 to a mass relative to the Sun's mass.

Turn-off point: from mass to age

The lifetime t of the star's main sequence phase depends on its luminosity and on its mass.

- A star with high luminosity burns more hydrogen each second than a star with low luminosity. So, the mass of a star with high luminosity decreases faster than the mass of a star with low luminosity and the lower the luminosity, the longer the star can burn.

- For two stars with different masses, the heavier star has more material to burn. So we see that the star's lifetime is directly proportional to its mass and inversely proportional to its luminosity.

Using the mass-luminosity relation, we find the lifetime as a function of the mass:

$t \propto \boldsymbol{M}^{-2.8}$

Task 16

? Take the mass derived in task 15 and estimate the age of the globular cluster relative to the estimated age of the Sun when it will leave the main sequence, 8.2×10^{9} years.

In conclusion, the whole Universe must be older than the age found in task 16.

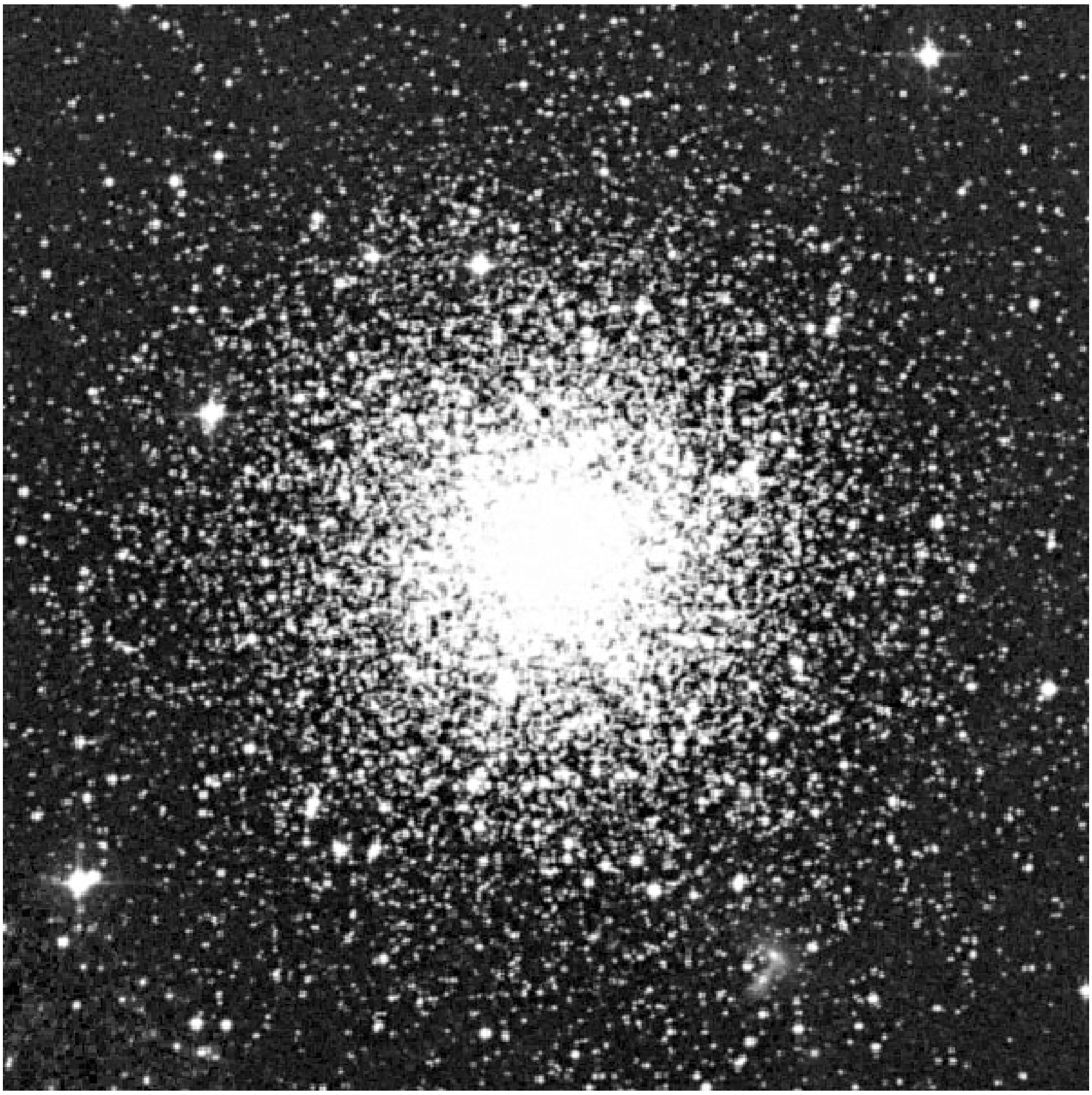

Figure 13: Overview image of globular cluster
This image shows M12. Each side of the image corresponds to 0.25 degrees (from the Digitized Sky Survey).

Determination of the diameter

To determine the diameter of M12, we need to know the angular diameter of M12. In Fig. 13 there are many stars at the centre of the cluster. Discuss which stars belong to the outer region of the cluster.

Task 17

? Measure the angular diameter, a, of the cluster M12 in centimetres and convert it to radians (see Mathematical Toolkit).

? Then calculate the diameter, d (see the small-angle approximation in the Mathematical Toolkit, page 8).

For the distance use either your own derived value or the value found by scientists of D = 4900 parsecs.

Determination of the total number of stars

To estimate the total number of stars, N, in the globular cluster, we need to make some assumptions:
1. The cluster consists of a mixture of all types of stars, but we assume that the average star is a Sun-like star, i.e. the absolute magnitude of a single star is about the same as that of the Sun.
2. We assume that each star contributes its total luminosity to the overall total luminosity of the whole cluster. In reality dust or other stars may occult some stars partially or fully.

Task 18

The absolute magnitude of M12 is given by $M_{cl} = -7.32$
The total luminosity of the cluster in terms of the Sun's luminosity is calculated from

$L_{cl}/L_{sun} = 2.512^{(M\_sun-M\_cl)}$

Remember: $M_{sun} = 4.8$.

As $L_{cl} \approx N \cdot L_{sun}$ and using assumption 1, the value for L_{cl}/L_{sun} is equal to N.
However, as a result of assumption 2, we would expect the real value of N to be a bit higher than L_{cl}/L_{sun}.

Additional Tasks

Further Reading

Scientific Papers

- de Bruijne, J.H.J., Hoogerneerf, R., and de Zeeuw, P.T., 2001, A&A, 367, 111–147: *A Hipparcos study of the Hyades open cluster.*

- Cragin, M., Lucyk, J., Rappaport, B.: *The Deep Sky Field Guide to Uranometria 2000.0*, 1993-96, Willmann-Bell, Inc.

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Read more about interstellar extinction in:
http://www.astro.virginia.edu/class/hawley/astr124/ism.html
http://tesla.phys.unm.edu/a111labs/cepheids/mags.html#Red

See also the Links on:
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Exercise 4: Measuring a Globular Star Cluster's Distance and Age
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Quick Summary

We measure blue (m_B) and green (visual, m_V) magnitudes of selected stars in the outer regions of a globular cluster shown on VLT images, convert the (m_B-m_V) values into stellar surface temperatures (T) and plot the m_V values as a function of the T values on a Hertzsprung-Russell diagram. The cluster's Main Sequence, seen in the plotted diagram, is compared with a distance-calibrated standard Main Sequence from the nearby Hyades cluster. The distance to the cluster can be determined by Main Sequence fitting and using the distance modulus. The cluster's age, which incidentally places a lower limit on the age of the Universe, can be estimated from the position of the turn-off point on the Main Sequence.

This teacher's guide contains solutions to the problems, with comments and discussion of any approximations and simplifications that have been made, and also additional considerations on the life cycle of stars. The aim is to maximise the usefulness of the exercise and to assist the teacher in preparing a teaching plan.

More on the Life of Stars

The lifetime of a star is the length of time it stays on the main sequence. We estimate the Sun's lifetime and then the lifetime of a star relative to the Sun's lifetime.

A protostar is formed from interstellar matter. Typically this interstellar matter consists of 74% hydrogen, 25% helium and 1% heavier elements.
When the interior temperature of a protostar reaches a few million Kelvin it can begin to burn hydrogen and become a main-sequence star.

Four hydrogen atoms fuse to form one helium atom. As the mass of one helium atom is only 99.3% of four hydrogen atoms, the residual mass (0.7%) is converted to energy.
For each kg of stellar matter, 0.007 kg will be converted to energy. From Einstein's law ($E=\boldsymbol{M}c^2$), we calculate the converted energy as 6.3×10^{14} J/kg. (c is the speed of light, 3×10^8 m/s).

The Sun's luminosity is $L_{sun}=3.85 \times 10^{26}$ W (W = J/s). From this we can calculate the mass of hydrogen fused each second:
$\Delta \boldsymbol{M} = 3.85 \times 10^{26} / (6.3 \times 10^{14}) = 6.11 \times 10^{11}$ kg/s

The star will leave the main sequence once about 11% of the hydrogen-mass has fused as the star's core then becomes unstable.

Taking the total mass of the sun to be $\boldsymbol{M}_{sun} = 2.0 \times 10^{30}$ kg we estimate the possible mass of hydrogen that can be fused during the star's lifetime as:
$0.11 \times 0.74 \times 2 \times 10^{30} = 1.6 \times 10^{29}$ kg.

Dividing this mass by the mass loss per second, we estimate the total lifetime of the Sun on the main sequence to be:
2.6×10^{17} s = 8.2×10^9 y, 1 y = $365 \times 24 \times 60 \times 60$ s = 3.15×10^7 s (or more than 8 billion years).

Observations of the Sun show that it is about 4 billion years old, so that it can expect a further 4 billion years or so of life on the main sequence.

Knowing the lifetime of the Sun, we can calculate the lifetime of any other star in terms of the Sun's lifetime.
The lifetime of any star depends on its mass. We will simplify the complex arguments to obtain a simple, but adequate formula:

The supply of hydrogen to fuel a star is proportional to its mass and t is inversely proportional to its luminosity, so: $t \propto \boldsymbol{M}/L$

The rate at which a star expends its energy increases rapidly with its mass. The experimental result for main sequence stars is given roughly by: $L = \boldsymbol{M}^{3.8}$, the so-called mass-luminosity relation. The exponent 3.8 is a compromise. It applies approximately to the medium range of stellar masses (0.5 ... 10) $\boldsymbol{M}_{sun}$.

So in conclusion we have (approximately): $t \propto \boldsymbol{M}/L = \boldsymbol{M}/\boldsymbol{M}^{3.8} = \boldsymbol{M}^{-2.8}$; we see that high mass stars evolve much faster than the Sun and low mass stars much more slowly.

Some examples:

A high mass star of about 10 solar masses will have a lifetime of only about $t = 0.0016\ t_{sun}$, or about 13 million years.

A low mass star of about 0.6 solar masses will have a lifetime of about $t = 4.2\ t_{sun}$, or 34 billion years. This is much greater than the age of the Universe itself. Therefore no low mass star in the Universe has yet completed its time on the main sequence.

Star sample selection

The globular cluster M12 contains about 150,000 stars. The image used in this exercise was obtained with FORS1 at ANTU (UT1 of the VLT). It covers only a small region in the outer parts of the cluster, chosen so as to avoid the most 'crowded' parts of the cluster where the stars appear to overlap. We have selected 45 stars that are representative of the cluster population. This sample size means that the workload is reasonable and that the students' measurements will be comparable with the scientific results, which were based on a much larger sample of stars. An image of M12, taken from the Digitized Sky Survey (DSS), was used for the additional tasks.

Analysing the image

We suggest that each group uses a transparency with the gauge on it. We have included the gauge on each picture so that it is possible to check that copying has not altered the scale of the picture and the students should first check that their transparent gauge matches the one on the image.

We suggest that the work is divided among groups of students and we have divided the image into six parts (training, calibration, A, B, C and D). The magnitudes are given for the five training stars. These five stars can be used to practise using the gauge to obtain accurate and repeatable results. The four calibration stars can then be measured by each group and used to calibrate the measurements between the groups.

To reduce errors we suggest that each star should be measured at least twice by each group and the results averaged.

It is very important to practise with the gauge before beginning the actual measurements. Measuring is not just putting the gauge over the image! For example, a star of magnitude 18.5 should be totally within the appropriate circle, but the surrounding sky should just touch the circle. The students should measure each star in this way. If the measurements are consistently too low or too high, then a correction can be made by adding or subtracting a constant as appropriate.

Fig. 3 in the Astronomical Toolkit is used to convert the B-V colour index to temperature.

A set of tables that can be printed out is provided, but the use of a spreadsheet program (for example, Excel) is recommended to simplify the calculation and display of the B-V colour index.

Task 1-8

The scientists' values as well as our own measurements are provided in a table (see Fig. 1).

	Scientists' Values				ESA/ESO's measurements/calculations			
Star	**B**	**V**	**B-V**	**T**	**B**	**V**	**B-V**	**T**
1	18.82	17.98	0.84	5250	18.70	17.90	0.8	5403
2	19.02	18.31	0.71	5744	19.00	18.20	0.8	5403
3	19.32	18.65	0.67	5864	19.30	18.70	0.6	6122
4	19.96	19.25	0.71	5699	19.90	19.10	0.8	5403
5	21.05	20.21	0.84	5265	21.00	20.10	0.9	5076
6	18.94	18.12	0.82	5348	19.00	18.20	0.8	5403
7	19.80	19.10	0.70	5757	19.80	19.20	0.6	6122
8	19.06	18.34	0.72	5702	19.00	18.40	0.6	6122
9	19.20	18.53	0.67	5844	19.10	18.50	0.6	5122
10	18.99	18.25	0.74	5614	19.00	18.20	0.8	5403
11	20.07	19.34	0.73	5620	20.10	19.40	0.7	5751
12	17.32	16.37	0.95	4918	17.20	16.40	0.8	5403
13	19.18	18.52	0.66	5884	19.10	18.50	0.6	6122
14	19.53	18.83	0.70	5722	19.60	18.80	0.8	5403
15	20.33	19.60	0.73	5639	20.30	19.50	0.8	5403
16	19.31	18.62	0.69	5792	19.30	18.60	0.7	5751
17	18.57	17.69	0.88	5140	18.70	17.80	0.9	5076
18	18.95	18.15	0.80	5405	18.90	18.10	0.8	5403
19	17.48	16.56	0.92	5012	17.50	16.60	0.9	5076
20	19.66	18.96	0.70	5738	19.60	18.80	0.8	5403
21	19.77	19.08	0.69	5792	19.80	19.00	0.8	5403
22	19.52	18.84	0.68	5818	19.50	18.80	0.7	5751
23	19.50	18.79	0.71	5734	19.50	18.90	0.6	6122
24	18.23	17.34	0.89	5122	18.30	17.40	0.9	5076
25	21.08	20.26	0.82	5345	21.10	20.20	0.9	5076
26	19.04	18.28	0.76	5552	18.90	18.20	0.7	5751
27	18.76	17.89	0.87	5160	18.80	18.10	0.7	5751
28	18.88	18.05	0.83	5309	18.90	18.10	0.8	5403
29	18.27	17.40	0.87	5183	18.30	17.40	0.9	5076
30	18.14	17.28	0.86	5189	18.20	17.30	0.9	5076
31	19.84	19.14	0.70	5783	19.80	19.10	0.7	5751
32	18.62	17.76	0.86	5197	18.60	17.80	0.8	5403
33	19.92	19.22	0.70	5725	19.90	19.20	0.7	5751
34	20.53	19.75	0.78	5487	20.40	19.70	0.7	5751
35	18.82	17.99	0.83	5300	18.80	18.00	0.8	5403
36	18.95	18.19	0.76	5511	18.80	18.20	0.6	6122
37	19.33	18.65	0.68	5812	19.30	18.70	0.6	6122
38	20.53	19.76	0.77	5502	20.50	19.60	0.9	5076
39	19.92	19.21	0.71	5734	19.90	19.20	0.7	5751
40	19.29	18.62	0.67	5861	19.30	18.70	0.6	6122
41	17.91	17.00	0.91	5026	18.00	16.90	1.1	4479
42	19.19	18.50	0.69	5789	19.20	18.50	0.7	5751
43	19.42	18.74	0.68	5831	19.30	18.70	0.6	6122
44	19.36	18.69	0.67	5841	19.30	18.70	0.6	6122
45	18.12	17.24	0.88	5145	18.20	17.20	1.0	4768

Figure 1: Solutions for Task 1 to Task 8
The table provides the star numbers and B, V, B-V and T values found by the scientists. Our own measurements are also indicated.

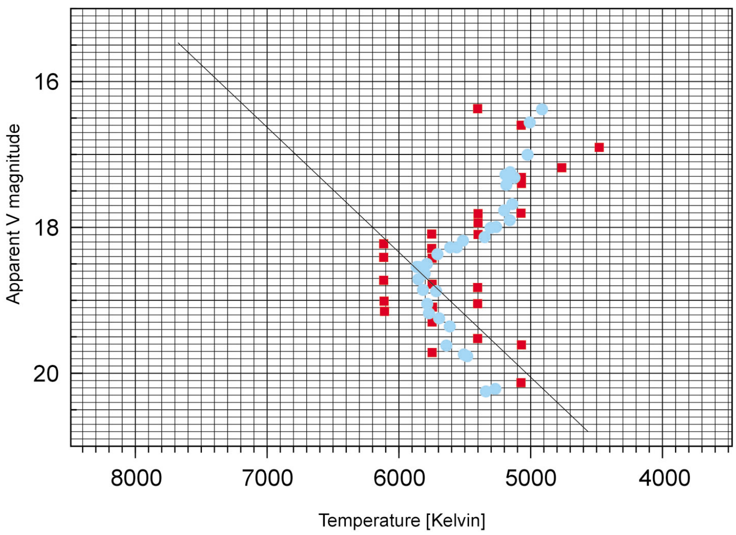


Figure 2: Hertzsprung-Russell Diagram of M12
The diagram shows our measurements (red) as well as the results obtained by the scientists (blue).

Task 9-13

The lower part of the diagram (Fig. 3) is quite short and the results are very sensitive to the slope of the best-fit line drawn between the data points. To simplify the process and avoid disappointing results, we have assumed that the shape of the main sequence is roughly the same for all star clusters, whatever their age, and so that all main sequences are parallel. Hence we can use the slope of the reference main sequence from the Hyades cluster as a guide.

The value of D depends on the position of the main sequence line in the cluster diagram.

Harris gives m_V-M_V = 14.02 for M12. We measured **13.9**.

Harris gives the value of D_{cl} = 4.9 kpc. This value is obtained by including the interstellar extinction between us and M12 (0.57 magnitudes) in the distance equation for M12, so m-M = 5 log D – 5 + 0.57.

We calculated $D=10^{(m-M+5)/5} = 10^{3.78}$ = **6.026 kpc** without interstellar extinction correction and $D= 10^{(m-M-0.57+5)/5} = 10^{3.666}$ = **4.634 kpc** with interstellar extinction correction.

For the following calculations we use the extinction corrected distance, 4.634 kpc.

Task 14-16

In our measurements a star at the turn-off point has an apparent magnitude of **18.7**. Scientists have measured the turn-off point to be 18.3 (Rosenberg et.al.).

We calculate now

$(L_{cl}/L_{sun}) = (D_{cl}/D_{sun})^2 \cdot (I_{cl}/I_{sun})$

Calculation of ratio (I_{cl}/I_{sun}):
As I_{sun} is much larger than I_{cl}, the ratio will be a very small number, so we suggest calculating I_{sun}/I_{cl} and then taking the reciprocal value for further calculation.

$(I_{sun}/I_{cl}) = 10^{(m\_cl\ -\ m\_sun)/2.5} = 10^{(18.7\ -\ (-26.5))/2.5} = 10^{18.08} = 1.202 \times 10^{18}$
so $(I_{cl}/I_{sun}) = 8.318 \times 10^{-19}$

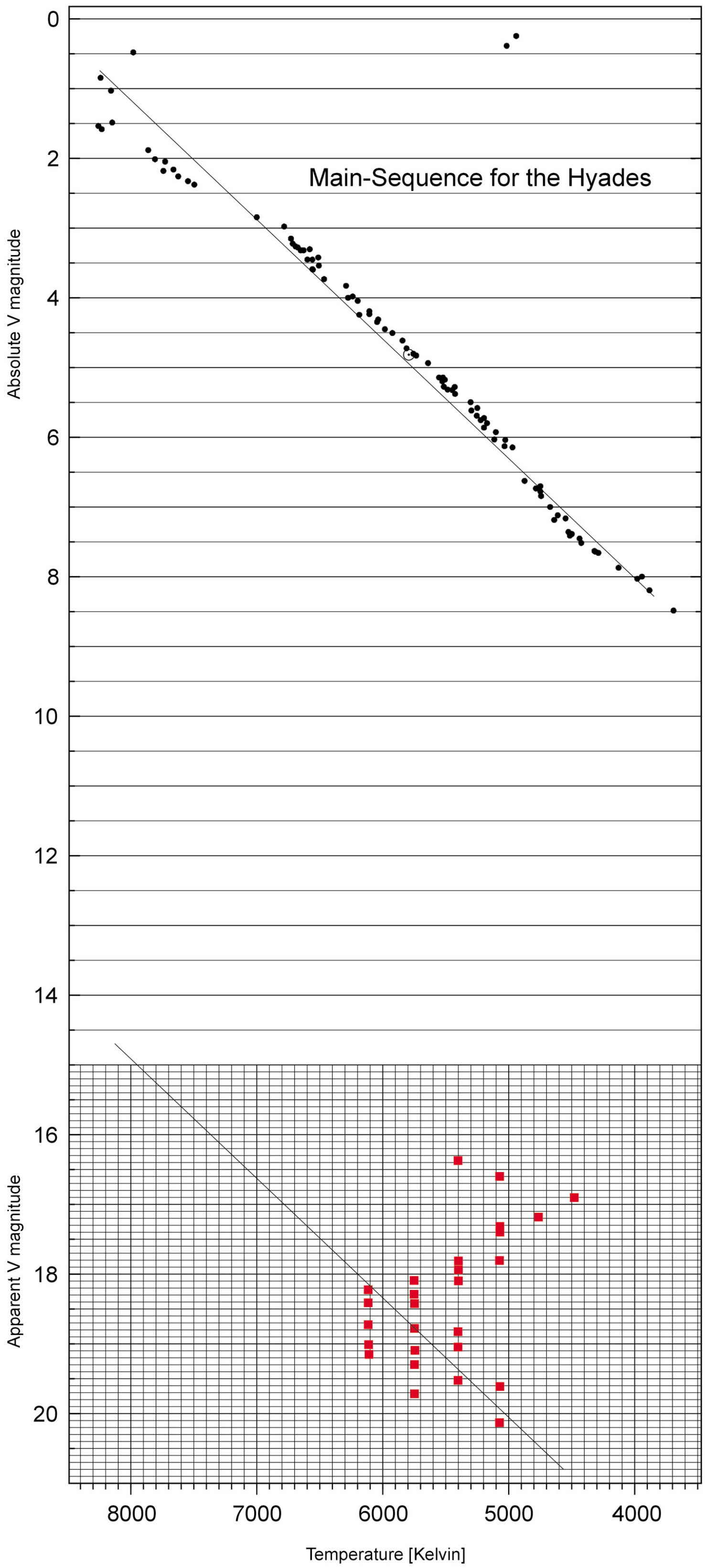


Figure 3: Hertzsprung-Russell Diagram of M12 and the Hyades cluster
This diagram includes the H-R diagram for the Hyades (upper part) as well as that of M12, using the authors' values. The lines represent the interpolation of the stars on the Main Sequence.

Further calculations:
$(D_{cl}/D_{sun}) = (4634 \times 3.086 \times 10^{13}) \;/\; 1.498 \times 10^{8} = 9.559 \times 10^{8}$

$(L_{cl}/L_{sun}) = (D_{cl}/D_{sun})^2 \times (I_{cl}/I_{sun}) = (9.559 \times 10^{8})^2 \times 8.318 \times 10^{-19} = \mathbf{0.76}$

$(\boldsymbol{M}_{cl}/\boldsymbol{M}_{sun}) = (L_{cl}/L_{sun})^{1/3.8} = \mathbf{0.93}$

$(t_{cl}/t_{sun}) = (\boldsymbol{M}_{cl}/\boldsymbol{M}_{sun})^{-2.8} = 1.224$

$t_{cl} = 1.224 \times t_{sun} = 1.224 \times 8.2 \times 10^{9} = \mathbf{10 \times 10^{9}}$ **years**

An alternative and somewhat simplier method to determine cluster ages exists. It's origin is empirical (based on measurements) and therefore less intuitive. It is to apply the following observed relation:

$M_V(TO) = 2.70 \log_{10}(t) + 1.41,$

where $M_V(TO)$ is the absolute magnitude of the turn-off point and t the age of the cluster in billions of years. Since
$M_V(TO) = m_V(TO) - (m_V(TO) - M_V(TO)\,) = m_V(TO) - (m_V - M_V)$
(the distance modulus is the same for the entire cluster), we get :

$m_V(TO) - (m_V - M_V) = 2.7 \log_{10}(t) + 1.41,$

which reduces to:

$t = 10^{[(m\_V(TO) - (m\_V - M\_V)) - 1.41) / 2.7]}$

Resulting ages by calculating different sets of turn-off magnitude and distance by using the originally proposed method and by using the alternative method described above:

Measured Turn-off magnitude [mV]	Calculated distance [pc]	Age, method 1 [billion years]	Age, method 2 [billion years]
18.7	4634	10.0	18.0
18.85	4634	11.1	20.5
18.5	4634	8.8	15.2
18.3	**4900**	7.0	11.6
18.3	4634	7.7	12.8
18.3	4500	8.0	13.5
18.7	6026 (no extinc.)	6.8	18.0

Bold figures are the best estimates from the literature.

Different methods for determining the age of globular clusters are described by Chaboyer et. al, who find ages in the range between **11.5 × 10^9 years** and **15.9 × 10^9 years** for M12.

Additional Tasks

Task 17

	cm	degrees	radians
Full image	14.8	0.25	
Angular Diameter, a	13.0	0.22	3.833×10^{-3}

$d = D_{cl} \cdot a = 4634 \times 3.833 \times 10^{-3} =$ **17.76 pc**

The cluster ends, when its density of stars reaches the density of background stars.

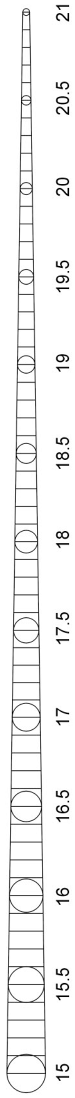
The value of the angular diameter, a, corresponds to $0.22 \times 60 =$ **13.2 arcminutes**. In the Uranometria 2000.0 Atlas the angular diameter is quoted as 14.5 arcminutes.

Task 18

$L_{cl}/L_{Sun} = 2.512^{(M\_sun-M\_cl)} = 2.512^{4.8-(-7.32)} \sim$ **70,500**

The total number of stars in the M12 is about 150,000 +/- 35,000 stars according to Carl Grillmair (SIRTF Science Center, private communication, 2002).

Figure 4:
Measurement gauge
This gauge needs to be copied onto transparencies and used for the measurements in tasks 1 to 6.



Teacher's Guide

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